Statistical Approach to Model the Deep Draft Ships' Squat in the St. Lawrence Waterway

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Abstract: In shallow waterways such as the St. Lawrence River, an accurate prediction of the squat is important to ensure a balance between the security and the efficiency of traffic. The Canadian Coast Guard is now studying the squat phenomenon and considering to reassess the actual underkeel clearance standards of the St. Lawrence Waterway. Hence, a field campaign was conducted with 12 deep draft ship sailings, during which the maximal squat was measured with on-the-fly global positioning system. All the variables that may influence the squat (speed, draught, water level, etc.) were also measured. Twenty of the empirical models that are used in practice to predict the squat were tested and the Canadian Coast Guard recommended to either optimize these models or develop new models. Therefore, statistical approaches to model the squat of deep draft ships that navigate on the St. Lawrence Waterway are proposed in this paper. The Eryuzlu model, which is presently used by the Canadian Coast Guard, was optimized by modeling its errors with a stepwise regression. New models were also developed with the regression tree technique. The performance of the statistical models was better than 10 empirical models that are considered the most suitable to predict the maximal squat in the St. Lawrence Waterway. The models built by regression tree gave the best predictions.

DOI: 10.1061/(ASCE)WW.1943-5460.0000003

CE Database subject headings: Ship motion; Waterways; Regression models; Statistics; St. Lawrence River; Shallow water.

Introduction

When a ship is moving through water, it has a position that is different from when it is at rest. This phenomenon is called squat. It is caused by the acceleration of water as it flows past the ship, which decreases the pressure on the ship's hull. It is accentuated when the ship is traveling through relatively shallow or confined waters (Stocks et al. 2002). In fact, the squat may cause the ship to reach the waterway bottom even though it has a sufficient clearance at rest. An accurate prediction of the squat is important to ensure enough maneuverability and a safety margin, since there is a risk to strike the waterway bottom. In Barrass (2004), recent ship groundings due to an excessive squat are related. In shallow water, an accurate prediction of the squat also ensures efficiency of the traffic as the ship can carry maximal charges and stay safe.

The main factor governing the squat is the ship's speed relative to the water. There are plenty of other factors that influence the ship squat. Some analytic expressions for the ship squat were developed and are presented in Gourlay (2000, 2006), and Varyani (2006). These articles also relate the early analytic methods. In practice, theoretical expressions are not handy for the routine use of mariners and marine traffic service officers. Hence, many researchers have developed empirical formulas based on model test results. In the next section, these models are listed and briefly presented. These models are not universally applicable for all types of ships, all channel shapes, and all speeds as they were built in experimental conditions. In MacPherson (2002), it is shown that the squat predictions obtained from the various models are different. It can be very confusing to choose the right model. These equations do not systematically take into account the same variables and, hence, the completeness of these models is different. They also vary according to the data sets that were used to build them and, thus, each model has its own restrictions. Finally, the squat models do not take into account a random error component. Otherwise, most models give conservative predictions (overpredicting) that reduce the net underkeel clearance estimation.

The St. Lawrence Waterway is an important gateway to the North American continent and has a lot of traffic (20,000 annual vessel movements and 40% of tonnage handled in Canada transits in the St. Lawrence Waterway). Simard (1982) developed a model based on Canadian field data to predict the squat in the St. Lawrence Waterway. Eryuzlu and D'Agnolo (1991) and Eryuzlu et al. (1994) proposed squat equations based on laboratory studies. The equations presented by Simard (1982), Eryuzlu and D'Agnolo (1991), and Eryuzlu et al. (1994) were considered the

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Note. Discussion open until October 1, 2009. Separate discussions must be submitted for individual papers. The manuscript for this paper was submitted for review and possible publication on February 7, 2008; approved on September 24, 2008. This paper is part of the ***Journal of Waterway, Port, Coastal, and Ocean Engineering***, Vol. 135, No. 3, May 1, 2009. ©ASCE, ISSN 0733-950X/2009/3-80–90/$25.00.

most appropriate techniques for the St. Lawrence Waterway (Morse and Simard 1991; Morse et al. 2004). The Canadian Coast Guard (CCG) has adopted a model developed by Eryuzlu and D'Agnolo (1991) to predict the deep draft squat in the St. Lawrence Waterway. The underkeel clearance standards set up by the CCG on the St. Lawrence River are based on this formula. The CCG is now considering to reassess the actual underkeel clearance standards of the St. Lawrence Waterway and to study the squat phenomenon. Better squat predicting models are highly desirable for an effective and secure underkeel clearance management.

CCG (2007) verified the goodness of fit of various squat models presented in the literature for the St. Lawrence Waterway. They found that the model adopted by the CCG for squat prediction developed by Eryuzlu and D'Agnolo (1991) was among the most accurate. Nevertheless, they also pointed out that even the best models could perform better. They recommended to optimize these models or to develop new models to ensure a better squat estimation. The objectives of the present work then are to optimize the most adequate model to predict the squat in the St. Lawrence Waterway and to propose a new potential approach to model the squat. The errors of the model developed by Eryuzlu and D'Agnolo (1991) are modeled with a stepwise regression to increase its performance. Furthermore, the regression tree approach is used to build a new squat prediction model.

The remainder of this article is organized as follows. First, the squat models that were found the most suitable to predict the squat in the St. Lawrence Waterway are presented. Second, the field measurements and the database are described. Third, the methodology is explained, and the results are presented in the following section. Finally, a discussion and the conclusions are presented.

Background

CCG (2007) evaluated the performance of various empirical models presented in the literature to predict the squat of three types of ships (bulk carrier, tanker, and container ship). Even though the conditions of application of the different equations were not always respected, all models were used to predict the squat. The performances of the different models were assessed by comparing the predicted values to the values measured by on-the-fly global positioning system (OTF-GPS) and the results were found unsatisfactory. Their recommendations were to refine the squat models to reduce both the quadratic errors and the bias or to develop new formulas giving a better performance. The squat models that were found the most suitable to predict the squat in the St. Lawrence Waterway are listed here and briefly explained. The purpose of this study is not to review all the squat models that were presented in the literature. For an exhaustive review of all the squat models, the reader is referred to Dick et al. (1991), Herreros Sierra et al. (2000), Stocks et al. (2002), Morse et al. (2002), Briggs et al. (2004), and Briggs (2006).

Simard Model

This model was developed for any types of ships that take the St. Lawrence Waterway (Simard 1969, 1982; Morse and Simard 1991). It was based on theoretical considerations of the principle of conservation of energy and was calibrated by field measurements in the St. Lawrence Waterway. It can be applied when the speed through the water is between 8 and 15 knots. It is recommended especially for restricted channels. It can be expressed as

$$\hat{S} = \frac{V^2}{2g} \cdot \left[\left(\frac{1.01}{1 - A_s/A_c} \right)^2 - 0.80 \right] \tag{1}$$

$$A_s = T \cdot b \tag{2}$$

$$A_c = h \cdot \min\{B, W\} \tag{3}$$

$$B = 10 \cdot b \tag{4}$$

where $\hat{S}$=predicted squat (m); V=ship speed relative to the water (m/s); g=acceleration of gravity (9.81 m/s^2); A_s=ship underwater cross-sectional area (m^2); A_c=cross-sectional channel area (m^2); T=ship draft (m); h=water depth (m); b=ship beam (m); B=restricted channel width (m); and W=channel width (m).

Barrass Model

Barrass (1979) developed this model for confined water or open water. It is an empirical model to predict the maximum squat (position undefined). It is applicable for any types of ships and when $1.1 < h/T < 1.5$. The following equation can be used:

$$\hat{S} = \frac{C_b \cdot S_2^{2/3} \cdot V_k^{2.08}}{30} \tag{5}$$

$$S_2 = \frac{A_s}{A_w} \tag{6}$$

$$A_w = A_c - A_s \tag{7}$$

where V_k=ship speed (knots); S_2=channel blockage; C_b=ship block coefficient; and A_w=net underwater channel cross-sectional area (m^2).

Norrbin Model

Norrbin (1986) proposed this model. It can be used to estimate the squat at the bow in open water. It is restricted to a Froude number smaller than 0.4. It is given by

$$\hat{S} = \frac{C_b}{15} \cdot \left(\frac{1}{L_{pp}/B} \right) \cdot \left(\frac{1}{h/T} \right) \cdot V_k^2 \tag{8}$$

where L_{pp}=ship length between perpendiculars (m).

Römisch Model

Römisch (1989) developed this model to predict the squat both at the bow and at the stern of the ship for a restricted or unrestricted channel having a rectangular or a trapezoidal shape. It is applicable when $1.19 < h/T < 2.25$. It is given by

$$S = \begin{cases} C_V \cdot C_F \cdot K_{\Delta T} \cdot T & \text{bow} \\ C_V \cdot K_{\Delta T} \cdot T & \text{stern} \end{cases} \tag{9}$$

where

$$C_V = 8 \cdot \left(\frac{V}{V_{cr}} \right)^2 \cdot \left[\left(\frac{V}{V_{cr}} - 0.5 \right)^4 + 0.0625 \right] \tag{10}$$

$$C_F = \left(\frac{10 \cdot C_b \cdot B}{L_{pp}} \right)^2 \tag{11}$$

$$K_{\Delta T} = 0.155\sqrt{\frac{h}{T}} \quad (12)$$

$$V_{cr} = K_c\sqrt{gh} \quad (13)$$

$$K_c = 0.2472 \cdot \ln\left(\frac{A_c}{A_s}\right) + 0.0241 \quad (14)$$

and V_{cr}=critical ship speed for unrestricted shallow water (m/s).

Millward Model

Millward (1992) presented this modified version of a model proposed in Millward (1990). It can be used for different types of ships transiting in open water, in channels with a width approximately equivalent to twice L_{pp} when $1.25 < h/T < 6.0$. The ship block coefficient ranges from 0.44 to 0.83. The formula for the bow squat can be expressed as

$$\hat{S} = 0.01 \cdot L_{pp} \cdot \left(61 \cdot 7C_b \cdot \frac{1}{L_{pp}/T} - 0.6\right) \cdot \frac{\mathsf{F}_{nh}^2}{\sqrt{1-\mathsf{F}_{nh}^2}} \quad (15)$$

where F_{nh} represents the Froude number.

Eryuzlu 1 (with Beam) Model

Eryuzlu and D'Agnolo (1991) introduced this empirical model. It is also presented in Eryuzlu et al. (1994). It was developed for a bulk carrier having a block coefficient of 0.8, a weight varying between 19,000 and 227,000 tons, and a static draught between 8.13 and 20.32 m. It is applicable for open water, a speed varying between 2 and 9 m/s and when $1.1 < h/T < 3.0$. It can be expressed as

$$\hat{S} = 0.181 \cdot \sqrt{T \cdot b} \cdot \left(\frac{V}{\sqrt{g \cdot T}}\right)^{2.269} \cdot \left(\frac{T}{h}\right)^{0.994} \quad (16)$$

This model is used by the CCG to predict the squat.

Eryuzlu 2 Model

Eryuzlu and D'Agnolo (1991) and Eryuzlu et al. (1994) presented this model. It has the same restrictions as the preceding one (Eryuzlu 1), but does not take into account the ship beam. It can be expressed as follows:

$$\hat{S} = 0.298 \cdot T \cdot \left(\frac{V}{\sqrt{g \cdot T}}\right)^{2.289} \cdot \left(\frac{T}{h}\right)^{0.972} \quad (17)$$

Eryuzlu 3 Model

Eryuzlu et al. (1994) presented this empirical model. It is based on the Eryuzlu 2 model with a factor for the channel width. This study was conducted for cargo ships and bulk carriers with bulbous bows and a block coefficient of at least 0.8. It can be used for restricted and unrestricted channels. It is applicable when $1.1 < h/T < 2.5$. The formula is for the squat at the bow

$$\hat{S} = 0.298 \cdot T \cdot \left(\frac{V}{\sqrt{g \cdot T}}\right)^{2.269} \cdot \left(\frac{T}{h}\right)^{0.972} \cdot K_b \quad (18)$$

$$K_b = \begin{cases} \frac{3.1}{\sqrt{W/b}} & \frac{W}{b} < 9.61 \\ 1 & \frac{W}{b} \geq 9.61 \end{cases} \quad (19)$$

where K_b is a width factor.

Japanese Model

This model was developed for the bow squat to respect the new standards regarding the waterways in Japan (Overseas Coastal Area Development Institute of Japan, unpublished work, 2002). The model is given by

$$\hat{S} = \left[\left(0.7 + 1.5 \cdot \frac{1}{h/T}\right)\left(\frac{C_b}{L_{pp}/B}\right) + 15 \cdot \frac{1}{h/T} \cdot \left(\frac{C_b}{L_{pp}/B}\right)^3\right] \cdot \frac{V^2}{g} \quad (20)$$

Field Measurements

The CCG led a field program in 2005, which is explained in detail in CCG (2007). This section presents a summary of the field measurements.

St. Lawrence Waterway

The St. Lawrence Waterway is an important commercial gateway to the North American continent and has a dense traffic. Nevertheless, it is also among the most difficult to navigate in the world. The 290 km waterway is extended from Montreal to Cap Gribane (Fig. 1). It is characterized by a complex flow regime extending from the outflow of Lake Ontario to the Gulf of St. Lawrence. It features a restricted and sinuous channel and dredging is necessary to maintain the depths available for navigation. There is 175 km of dredged channel. The maintained water depth in the dredged reaches varies from 10.7 to 12.5 m. The waterway is exposed to significant fluctuations in water levels due to tides as well as extreme currents and ice conditions.

Vessels

Twelve sailings were carried out on the St. Lawrence Waterway in 2005 with 10 different deep draft ships of three types (bulk carrier, tanker, and container ship). The selected ships are representative of the different types of ships that navigate on the St. Lawrence Waterway. The departure and arrival cities are presented in Fig. 1. Table 1 presents the itineraries of the 12 sailings and the characteristics of the ships that were used.

Squat Measurement

The vessel positioning was measured by OTF-GPS technology using the Canadian Coast Guard's GPS network. The observations taken with OTF-GPS can be very accurate and can be used as a basis to validate a prediction model of the ship's squat given the environmental conditions and operating parameters of the ship (Parker and Huff 1998; Dunker et al. 2002; Morse et al. 2004). Validation pretests confirmed that the vertical positioning obtained in the squat measurement environment could have an accuracy of ±5 cm in 95% of cases.

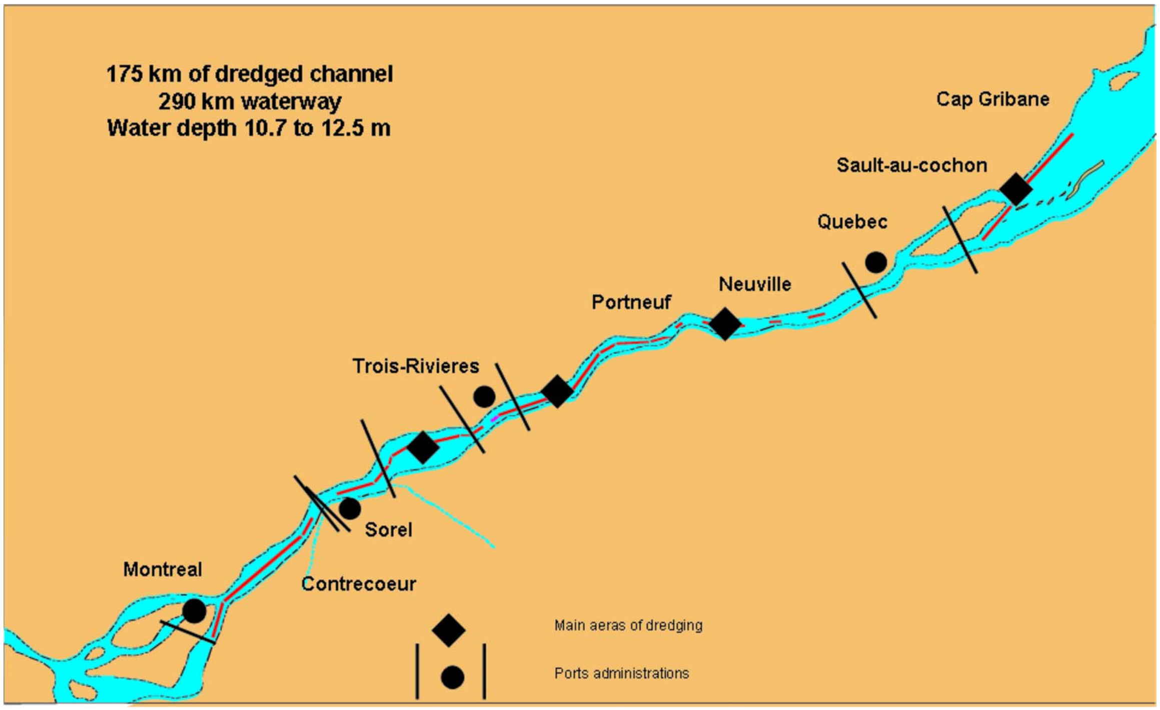

Fig. 1. Map of the St. Lawrence Waterway with the main areas of dredging and the departure and arrival cities of the 12 sailings: (1) Montreal–Portneuf; (2) Quebec–Trois–Rivieres; (3) Montreal–Neuville; (4) Portneuf–Montreal; (5) Neuville–Montreal; (6) Montreal–Neuville; (7) Sault-au-cochon–Quebec; (8) Quebec–Contrecoeur; (9) Quebec–Montreal; (10) Sault-au-cochon–Quebec; (11) Quebec–Sault-au-cochon; and (12) Sorel–Montreal

After adapting the OTF-GPS network to the needs of the squat study and validating the data collection methodology via various tests, an exhaustive field measurements campaign was carried out in the summer of 2005.

During the 12 sailings of the 2005 field program, each ship was escorted by two boats. The first one was located ahead of the ship. It was equipped with a OTF-GPS antenna, which allowed it to measure the water level and to provide a reference for the vertical position of the ship. Bathymetric records were also taken along the trajectory with this boat. Finally, this boat was used to measure indirectly the speed currents using a *speedlog*. The second one was located behind the ship. It was equipped with a recording device (*acoustic Doppler current profiler*) to measure the currents.

Eight complete OTF-GPS systems were used to have mobile stations on the vessel and the escort boat. Two antennas were

Table 1. Summary of the 12 Sailings and Characteristics of All the Ships Used in the Field Program of 2005 (CCG 2007)

Sailing number	Ship type	L/b[a] (m)	L_{pp}[b] (m)	T[c] (m)	C_b[d]	VSD[e] (metric tons)	Itinerary	Date
1	Container ship	294/32	282	10.0	0.70	69,065	Montreal–Portneuf	09/07/2005
2	Bulk carrier	225/32	217	11.0	0.87	86,035	Quebec–Trois–Rivieres	14/07/2005
3	Container ship	245/32	232	9.5	0.70	56,526	Montreal–Neuville	18/07/2005
4	Container ship	294/32	282	10.0	0.70	69,065	Portneuf–Montreal	25/07/2005
5	Container ship	202/31	195	10.0	0.69	43,805	Neuville–Montreal	29/07/2005
6	Container ship	173/27	165	8.9	0.60	27,245	Montreal–Neuville	31/07/2005
7	Tanker	274/48	264	15.6	0.82	171,554	Sault-au-cochon–Quebec	05/08/2005
8	Bulk carrier	200/24	191	10.6	0.90	43,941	Quebec–Contrecoeur	16/08/2005
9	Tanker	183/32	174	9.4	0.74	61,670	Quebec–Montreal	18/08/2005
10	Tanker	183/32	172	11.4	0.77	56,474	Sault-au-cochon–Quebec	26/08/2005
11	Bulk carrier	200/24	191	10.7	0.90	43,941	Quebec–Sault-au-cochon	01/09/2005
12	Bulk carrier	185/24	177	8.6	0.84	35,349	Sorel–Montreal	02/09/2005

[a] L and b represent, respectively, the ship length and width.
[b] L_{pp}=length of the ship between perpendiculars.
[c] T=ship draught.
[d] C_b=ship block coefficient.
[e] VSD=ship volume displacement.

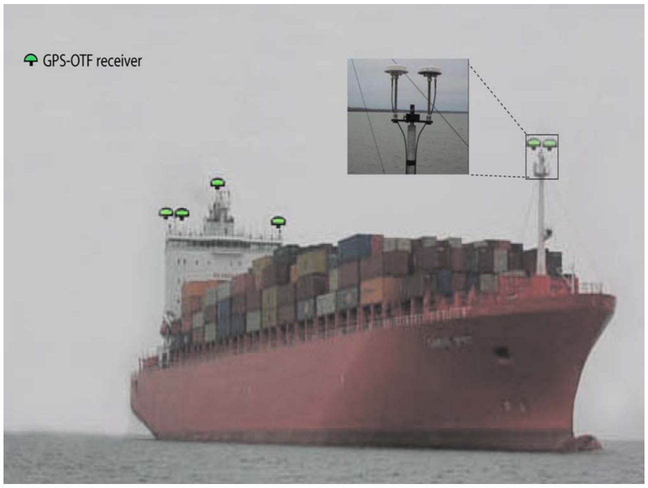

Fig. 2. Typical layout of OTF-GPS equipment aboard ships

installed on the longitudinal axis of the vessel (masts on the bow and the stern) and two others on the starboard and port sides (Fig. 2). This configuration (number and layout of antennas) ensures simultaneous measurements of all vessel movements (rolling, sinkage, trim, etc.).

A thorough filtering process was applied to the data from the OTF-GPS observations to validate the quality of positioning for calculating squat. The various processing and quality control methods applied enabled an average of 84% of the data to be retained. The data represent an average recording of about 120 h for each antenna. Considering the 12 transits, the field campaign in 2005 generated an imposing quantity of data, for which the number of reliable OTF-GPS observations exceeded 750,000 values.

Special attention was given to processing water level data, which affects squat values directly during field level campaigns. A spectral analysis, combined with a filtering process, was performed to eliminate the various forms of noise contaminating the data collected. Once validated, the water level data were used to determine the channel water depth along the transit route of the trucked vessels.

Finally, the current speeds measured with various instruments (ADCP, speedlog) were merged and then processed in the light of simulation results from a numerical hydrodynamic model (Hydrosim). The values thus obtained were associated with the vessel speed (SOG) calculated using OTF-GPS observations to deduce the vessel speed relative to water (SOW). This latter speed is the most representative of the hydrodynamic mechanisms governing the squat.

Table 2. Characteristics of the 10 Ships Used in the Field Study of 2005

Parameters measured	Minimum	Maximum
Ship draught (m)	8.6	15.6
Block coefficient	0.6	0.9
Ship width (m)	24	48
Ship length (m)	177	294
Ship length between perpendiculars (m)	165	282
Volumetric ship displacement (metric tons)	27,245	171,554

Table 3. Characteristics of the Channels Used in the 12 Sailings in the Field Program of 2005

Parameters measured	Minimum	Maximum
Width of restricted channel (m)	235	305
Water depth (m)	11.56	54.43
Current speed (m/s)[a]	2.254	−2.651
Ratio water depth/draught	1.1232	4.7533

[a]If positive, the current is in the same direction as the ship motion.

Calculating Squat from OTF-GPS Data

In the first instance, sinkage, defined as the vertical settlement of the antenna, was assessed for each antenna installed on the monitored vessel. The antenna sinkage value was determined by comparing antenna height relative to water surface in static and dynamic modes (vessel underway). This method enabled squat to be measured with an accuracy of ± 10 cm. Second, the squat caused by various vessel movements was determined by identifying the maximum sinkage measured on the four vessels' antennas. The maximum sinkage value was designated as the "vessel squat."

Once the vessel squat has been calculated over the entire transit, further analysis was carried out on chronological data series. In order to determine the squat that gives the maximum underkeel clearance (UKC) reduction, a periodic processing with a filter of 30 sec amplitude was applied to this series. For consistency, only maximum squat values were retained to create a new data matrix. The new squat values resulting from filtering were referred as the "maximum squat" of the vessel.

A preliminary validation of the maximum squat values was carried out based on the frequency of observation at each antenna during the same transit. A second squat data validation was done by examining squat behavior according to the most important factors affecting the UKC. These two validations yielded to the rejection of some doubtful data.

Database

The CCG analyzed the measured squat, the hydrographic data, the currents, and the ship descriptive data. Hence, the synthesized database has a total number of 5,785 observations measured during 12 sailings with 10 different deep draft ships and is presented in CCG (2007). The observations were taken in restricted and unrestricted channels and in different navigation conditions. The main characteristics (ships, channel, and navigation conditions) of the 12 sailings are summarized, respectively, in Tables 2–4.

Methodology

The recommendations given in Gharbi (2007) to either optimize the existing squat models or to develop new models were followed. Four data sets were created with the database: (1) all ship

Table 4. Characteristics of the Navigation Conditions during the 12 Sailings in the Field Program of 2005

Parameters measured	Minimum	Maximum
Squat maximum (m)	0.05	1.58
Speed relative to the water (m/s)	1.05	7.83
Speed over ground (m/s)	0.17	9.16
Froude number	0.0794	0.6361

Table 5. List of the Variables Used in the Study

Symbol	Variable description	Unit of measure
A_s	Ship underwater cross-sectional area	(m^2)
A_c	Cross-sectional channel area	(m^2)
A_w	Net underwater channel cross-sectional area	(m^2)
b	Ship beam	(m)
B	Restricted channel width	(m)
C_b	Ship block coefficient	
F_{nh}	Froude number based on the undisturbed water depth	
h	Water depth	(m)
h/T	Ratio water depth-ship draft	
L	Overall length of the ship	(m)
L_{pp}	Ship length between perpendiculars	(m)
PAR_h/T	Water depth effect	
S_2	Channel blockage	
st	Ship type (st=0 for all ship types, =1 for the container ship, =2 for the tanker, and =3 for the bulk carrier)	
T	Maximal ship draft	(m)
V	Ship speed relative to the water	(m/s)
V_{cr}	Critical ship speed for unrestricted shallow water	(m/s)

types; (2) container ship; (3) tanker; and (4) bulk carrier. All the explanatory variables that are used in the different squat models were listed. Some variables that are less pertinent or highly correlated to another variable that is already in the list were removed. For example, the restricted channel width was used instead of the channel width since it is more appropriate from a physical point of view. The explanatory variables used are listed in Table 5.

For comparison purposes, the performance criteria used were the same as in the work of CCG (2007): the mean-square-error (MSE), the coefficient of determination (R^2), and the mean relative bias (MRB). The MSE gives a measure of the deviation of the predicted values from the observed ones. When the MSE is weak, there is little difference between the predicted response values ($\hat{y}_i$) and the observed response values (y_i)

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2 \tag{21}$$

where n denotes the sample size. The R^2 allows us to verify the model fitting. When it is close to 1, the model is presumably good, but when it is close to 0, the model is probably not adequate

$$R^2 = 1 - \frac{\Sigma_{i=1}^{n}(\hat{y}_i - y_i)^2}{\Sigma_{i=1}^{n}(y_i - \bar{y})^2} \tag{22}$$

Finally, the MRB was used to verify if the model tends to overestimate or underestimate the observations. The mean relative bias provides the importance of the bias (%) from the observations

$$\text{MRB} = \frac{1}{n}\cdot 100 \cdot \sum_{i=1}^{n}\left(\frac{\hat{y}_i - y_i}{y_i}\right) \tag{23}$$

Optimization of the Eryuzlu 1 (with Beam) Model

The first approach we adopted was to model the errors of the Eryuzlu 1 model. We chose this model since it was adopted by the CCG to predict the squat in the St. Lawrence Waterway. The idea behind this approach was that the Eryuzlu 1 formula explains mostly the relation between the squat and the principal explanatory variables. Nevertheless, the introduction of some variables that are not included in the model could improve the squat predictions. To refine the Eryuzlu 1 formula, we modeled the errors using a stepwise regression (Myers 1986). This technique is advantageous when we need to choose which explanatory variables to include in the model and when there are a lot of explanatory variables that may or may not be relevant for making predictions about the response variable. The stepwise regression allows the selection of the explanatory variables that provide important information about the response variable. It consists of an automatic procedure, which step by step, adds or retrieves a predictive variable to the model based on a selection criterion. The Fisher test is the most common criterion to choose the variables and it was used in this study. At each step, the residual sum of squares of the actual model is compared to those of the next model with a Fisher test and the procedure includes/excludes a variable to the model. The selection ends when no more variables can enter or exit the model. Although this technique has been widely used, it must be noted that it has been criticized on many points (Hocking 1976). The predictive capacity of the model can be affected by the model selection (Hurvich and Tsai 1990). Furthermore, the usual Fisher test and the coefficient of determination used in the model selection are biased in the case of stepwise regression (Rencher and Pun 1980).

We first applied the Eryuzlu 1 model and computed the errors

$$E_{E1} = S - 0.181 \cdot \sqrt{T \cdot b} \cdot \left(\frac{V}{\sqrt{g \cdot T}}\right)^{2.269} \cdot \left(\frac{T}{h}\right)^{0.994} \tag{24}$$

where E_{E1} represents the errors of the Eryuzlu 1 model; S=maximal squat measured (m); T=ship draught (m); b=ship beam (m); V=ship speed relative to the water (m/s); g=constant (9.81 m/s^2); and h=water level (m). The starting model of the stepwise regression can be expressed as follows:

$$E_{E1} = \beta_0 + \varepsilon \tag{25}$$

where β_0=origin; and ε represents the residuals that are normally distributed with a 0 mean and a variance of σ^2. Depending on the data set, we added or retrieved the 17 predictive variables to the model. The full model has the form

$$E_{E1} = \beta_0 + \beta_1 F_{nh} + \cdots + \beta_{17}\text{st} + \varepsilon \tag{26}$$

where $\beta_i (i=1,\ldots,17)$ represent the coefficient of each independent variable. To obtain a squat prediction, we joined the error model and the Eryuzlu 1 model

$$\hat{S} = E_{E1} + 0.181 \cdot \sqrt{T \cdot b} \cdot \left(\frac{V}{\sqrt{g \cdot T}}\right)^{2.269} \cdot \left(\frac{T}{h}\right)^{0.994} \tag{27}$$

In the remainder of the text, this optimized model will be noted Eryuzlu OPT.

New Model Development

The second approach considered is the development of a new model. We used classical regressive techniques (stepwise regression, ridge regression) as a preliminary analysis. A variable selection with a stepwise regression is a poor strategy when some of the explanatory variables are correlated, as this was the case in this work. It is better to use a technique that takes into account

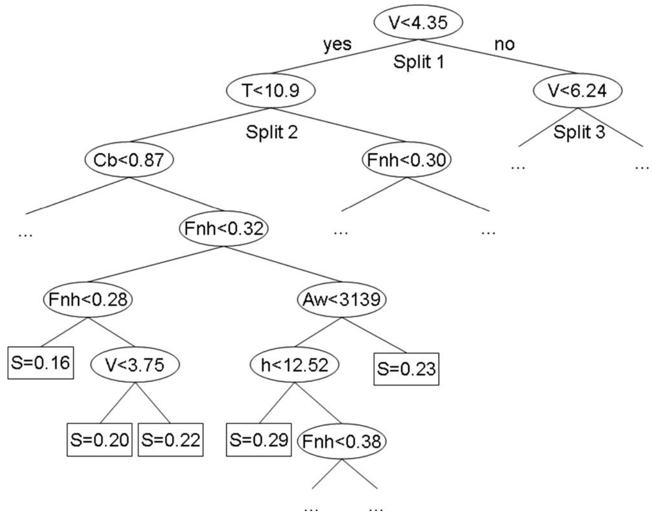

Fig. 3. Parts of a regression tree adjusted on the bulk carrier data set; the intermediate nodes are represented by an ellipse and the final nodes are represented by a rectangle

multicollinearity in the estimation of the regression coefficients such as the ridge regression (Marquardt and Snee 1975). However, even the ridge regression was not a good choice for the problem. The assumptions of the model were violated and, hence, it was not valid. The problem with the regression techniques is that they suppose a specific form for the relation between the squat and the explanatory variables. In reality, this relation seems much more complex and is obviously not linear. Consequently, a nonparametric technique based on the least squares such as the classification and regression tree (CART) is more suitable. This technique is presented in detail in Breiman et al. (1984). This technique belongs to the algorithmic modeling techniques, such as the artificial neural networks, for example. The general approach is that we have a black box inside of which the dynamics are complex and partly unknown. We have a set of observed values ($\mathbf{x}$) that represent the inputs to the box and a set of predicted values ($\mathbf{y}$), which represent the outputs of the box. The problem is to find an algorithm $f(\mathbf{x})$, such that for future values of $\mathbf{x}$, $f(\mathbf{x})$ will be a good predictor of $\mathbf{y}$ (Breiman 2001).

The CART algorithm consists of partitioning the data into a sequence of binary splits to terminal nodes, which give a predicted value. To obtain a predicted value, an $\mathbf{x}$ is dropped down the tree. At each intermediate node, the rule allows to determine whether to go left or right depending on the $\mathbf{x}$ values. It stops at a terminal node and the predicted value assigned is given by that node. Fig. 3 presents parts of a regression tree that were adjusted on the bulk carrier data set. The complete tree is too large to be presented. For example, if we drop the observed values $\mathbf{x}=\{V=3.5\ \mathrm{m/s}, T=10.6\ \mathrm{m}, C_b=0.9, \mathsf{F}_{\mathrm{nh}}=0.29, \ldots\}$ down this tree, the first question to be asked will be: is the speed smaller than 4.35 m/s? The answer will be yes and we will follow the left branch of the tree. The question at the next intermediate node is: is the ship draft smaller than 10.9 m? The answer will be yes again and we will follow the left branch. The following question will be: is the block coefficient smaller than 0.87? The answer will be no and then, we will go to the right. It continues like this until we reach the final node, which gives the squat prediction. According to these $\mathbf{x}$ values, the squat will be 0.20 m.

To construct a tree predictor, three elements are necessary: (1) a way to select a split at each intermediate node; (2) a rule for determining when a node is terminal; and (3) a rule for assigning a predicted value to each terminal node (Breiman et al. 1984). First, at each node, the split is chosen to minimize the MSE (it is the split that best separates the high response values from the low ones). The splitting is done regarding the explanatory variables that are either quantitative or categorical. For example, a split can take the form $\{\mathsf{F}_{\mathrm{nh}} \leqslant 0.3\}$ or $\{\mathrm{st}=1\}$. Second, a node is terminal when it is pure (all response values in the node are equal) or when the number of cases in the node is smaller or equal to the minimal node size (this value is arbitrary and depends on the number of observations). If this value is very small, the tree can take huge dimensions and the opposite statement is also true. For this work, we fixed the minimal node size to 10 values. Third, the predicted value assigned to each terminal node is the mean of the response values that belong to this node.

A sequence of trees is obtained $\{T_1, T_2, \ldots, T_{\max}\}$, with the last one being the full size tree or also called starting tree. The problem with the starting trees is that their size is not optimal (it can be huge) and the predicted values can be too optimistic. Hence, the starting tree describes the data set very well as all observations are used to build the tree, but its capacity to obtain predicted values for a new data set could be poor. A pruning process was then employed to find the right size tree, which respects both the accuracy and parsimony principles. The right size tree was chosen according to the relative mean squared error of each subtree in the sequence. To get honest estimates of the relative mean squared error, it was computed with a tenfold cross-validation. It consists of partitioning the sample into 10 random subsamples of equal size. The response values in each subsample are predicted with the remaining observations and the relative mean squared error is computed. This was repeated for all trees in the sequence. The smallest tree within one standard error of the minimum relative mean squared error was chosen as the predictor as suggested by Breiman et al. (1984). To lighten the text, the models developed with the regression tree will be noted RegT in the latter part of the paper.

Results

Table 6 presents the coefficients of each variable in the stepwise regression adjusted on the errors of the Eryuzlu 1 model. The coefficients of each variable are very different according to the data set used to adjust the model. There are six variables that are not significant (5% critical level) and, hence, 11 variables seem to significantly improve the models adjusted for all ship types and for the container ship. When the model is adjusted for the tanker and the bulk carrier, there are less significant variables that could improve the model. It is interesting to note that even though the ship speed is an explanatory variable of the Eryuzlu 1 model, it was included in the four Eryuzlu OPT models. The Eryuzlu 1 model could be modified to better represent the relation between the ship squat and the ship speed. Other variables could improve the Eryuzlu 1 model, such as the Froude number and the water depth effect that are significant in the four models. The normality, homoscedasticity, and independence hypothesis made when using a technique such as the stepwise regression are not respected in all cases.

The regression tree technique was used to model the squat logarithm. We preferred the squat logarithm to the squat simply because it gives a better performance. The first models are computed with the 17 explanatory variables listed in Table 5. In order to reduce the model without losing too much informa-

Table 6. Coefficients of Each Variable in the Stepwise Regression Model Adjusted on the Eryuzlu 1 Errors

	Regression coefficients			
Variable	All ship types	Container ship	Tanker	Bulk carrier
Intercept	2.7907	1.5143	7.7430	−10.5487
F_{nh}	0.9934	−3.5966	−0.4332	0.4941
C_b	−0.9074	0.0000[a]	−10.0913	0.0000[a]
S_2	−1.0178	0.0000[a]	−2.7606	−0.2990
V	−0.0544	0.3238	−0.0282	0.0000
T	−0.1451	−0.1121	0.0000[a]	−1.3506
b	−0.0809	−0.0170	0.0000[a]	−0.4912
h	0.0000[a]	0.2186	0.0000[a]	0.0000[a]
B	0.0000[a]	0.0000[a]	0.0000[a]	0.0000[a]
L_{pp}	0.0248	−0.0399	0.0000[a]	0.0000[a]
L	−0.0223	0.0401	0.0000[a]	0.1824
A_s	0.0043	0.0000[a]	0.0000[a]	0.0000[a]
A_c	0.0000[a]	0.0000[a]	0.0015	0.0000[a]
A_w	0.0000[a]	−0.0003	−0.0015	0.0000[a]
PAR_h/T	−0.0790	−0.9393	−0.3530	0.0613
V_{cr}	0.0000[a]	0.5155	0.0000[a]	0.0000[a]
h/T	0.0000[a]	−3.7300	0.0000[a]	0.0000[a]
st	0.0359	0.0000[a]	0.0000[a]	0.0000[a]

[a]Not significant (5% critical level).

tion, we retrieved other variables that do not bring an improvement to the model. We retrieved five variables from the list: the cross-sectional channel area (A_c), the ship underwater cross-sectional area (A_s), the restricted channel width (B), the water depth effect (PAR_h/T), and the overall length of the ship (L). The final models obtained are not shown because there are several final nodes and it is not visually interpretable. Nevertheless, a partition of the tree adjusted with the bulk carrier data set is shown in Fig. 3. Furthermore, the variables used in each model (at least one node is based on one of these variables) are presented in Eq. (28)

$$\hat{S}=\begin{cases} f(A_w,b,C_b,\mathsf{F}_{nh},h,h/T,L_{pp},S_2,\mathrm{st},T,V,V_{cr}) & \text{all ship types}\\ f(A_w,C_b,\mathsf{F}_{nh},h,S_2,T,V,V_{cr}) & \text{container ship}\\ f(A_w,C_b,\mathsf{F}_{nh},h,S_2,V,V_{cr}) & \text{tanker}\\ f(A_w,C_b,\mathsf{F}_{nh},h,h/T,S_2,T,V,V_{cr}) & \text{bulk carrier}\end{cases} \tag{28}$$

Table 7 presents the adjustment results obtained with Eryuzlu OPT models, those obtained with RegT with the 13 explanatory variables, and also those obtained with the best empirical models. This table shows that the performance criteria vary considerably according to the ship type. Modeling the errors improves the performance of the Eryuzlu 1 model. Eryuzlu OPT now surpasses the performance of the 10 empirical models. However, the performance of RegT is highly superior to the performance obtained with all other models. The coefficient of determination obtained with RegT varies between 0.84 and 0.91, while it ranges from 0.45 to 0.82 for the empirical models and from 0.78 to 0.83 for Eryuzlu OPT. For all cases, the R^2 obtained with RegT is better. For the MSE, RegT and Eryuzlu OPT are the most performant. Nevertheless, the MSE is good in general (between 0.00 and 0.11) with all models. The MRB obtained with RegT varies between 0.7 and 1.0 and between 1.8 and 3.9 for Eryuzlu OPT, which means that the squat is slightly overestimated with these two models. The MRB ranges from −36.1 to 128.7 for the 10 empirical models. According to the data set (all ship types, container ship, tanker, or bulk carrier) and the model used, the squat is either overestimated or underestimated. Most of the time, the models give a conservative prediction of the squat, an overestimation (positive MRB), except for the tankers. For this type of ship, it is mostly underestimated (negative MRB). When the Eryuzlu 1 model is applied to this data set, most values are underestimated. The underprediction of tanker squat is probably-due to the larger ratio restricted channel width/length of the ship (B/L) for these ships. Hence, the squat is proportional to the B/L ratio (Tuck 1966; ICORELS 1980). The Eryuzlu 1 model does not depend on this ratio, and then will tend to underpredict the squat for large B/L values. It should be noted that RegT is ad-

Table 7. Performance of the Eryuzlu OPT and RegT Models to Predict the Squat in the St. Lawrence Waterway by Comparison to the Best Empirical Models

	Performance criteria											
	All ship types			Container ship			Tanker			Bulk carrier		
Model	R^2	MSE	MRB	R^2	MSE	MRB	R^2	MSE	MRB	R^2	MSE	MRB
Eryuzlu OPT	0.80	0.01	3.9	0.78	0.01	3.7	0.83	0.01	1.8	0.83	0.00	2.4
RegT	0.91	0.01	0.9	0.84	0.01	1.0	0.89	0.01	0.8	0.89	0.00	0.7
Barrass[a]	0.54	0.03	18.0	0.66	0.02	6.0	0.80	0.03	−12.7	0.51	0.06	63.0
Eryuzlu 1[a]	0.62	0.03	3.1	0.65	0.02	9.0	0.77	0.05	−20.6	0.51	0.02	13.8
Eryuzlu 2[a]	0.56	0.03	−1.7	0.62	0.02	0.3	0.80	0.07	−28.0	0.45	0.02	17.5
Eryuzlu 3[a]	0.59	0.03	0.2	0.64	0.02	2.9	0.81	0.06	−25.8	0.48	0.02	18.2
Japanese[a]	0.66	0.03	19.3	0.56	0.02	−0.4	0.80	0.02	14.0	0.59	0.04	54.6
Millward[a]	0.45	0.09	51.4	0.63	0.04	23.4	0.82	0.02	13.1	0.46	0.21	127.8
Norrbin[a]	0.67	0.02	6.7	0.57	0.02	−8.7	0.81	0.02	−3.1	0.61	0.02	39.2
Römisch at the bow[a]	0.61	0.04	7.5	0.51	0.04	−18.7	0.66	0.07	22.0	0.72	0.02	36.1
Römisch at the stern[a]	0.53	0.04	−13.9	0.57	0.03	−10.7	0.63	0.11	−36.1	0.52	0.01	0.0
Simard[a]	0.59	0.03	0.7	0.64	0.02	1.0	0.79	0.07	−24.0	0.48	0.02	21.3

[a]Results presented in Bealieau et al. (2008).

Table 8. Performance of RegT Model Obtained by Leave-One-Out Cross-Validation

	Performance criteria											
	All ship types			Container ship			Tanker			Bulk carrier		
Model	R^2	MSE	MRB	R^2	MSE	MRB	R^2	MSE	MRB	R^2	MSE	MRB
RegT	0.89	0.01	1.3	0.80	0.01	1.1	0.87	0.01	1.2	0.85	0.00	0.9

justed to the squat logarithm, but the performance criteria are calculated after the inverse transformation (exponential).

The regression tree technique gives good results since this technique describes well the data used to fit the model. Nevertheless, the quality of the predictions can be verified with a leave-one-out cross-validation. It consists of taking a single observation in the sample as the validation data and to predict this observation with all remaining observations. This is repeated until all observations are used once as validation data. In Table 7, the performance criteria of RegT are not computed with the leave-one-out cross-validation to be able to compare the performance criteria with those obtained from the other models. However, it is interesting to compute the cross-validation errors to get honest estimates of the performance criteria. Table 8 presents the performance criteria of RegT obtained by the leave-one-out cross-validation. The performance is slightly affected by the cross-validation, but remains very good. RegT then seems to be a good squat predictor.

Summary and Conclusions

We used two new approaches to model the maximal squat of deep draft ships that transit in the St. Lawrence Waterway. We optimized the Eryuzlu 1 model by modeling its errors, and a new model was built using the regression tree technique. The database used was built with three types of ships: container ship, tanker, and bulk carrier. The relation between the squat and the explanatory variables seemed to depend on the type of ship. Hence, four data sets were created: all ship types combined as well as a data set for each type of ship.

We chose the Eryuzlu 1 model because the CCG uses it. Furthermore, it is one of the most appropriate techniques to predict the maximal squat on the St. Lawrence Waterway. Modeling its errors with a stepwise regression has improved the performance (R^2 and MRB) of the Eryuzlu 1 model on the four data sets (Table 7). Nevertheless, the results were not satisfactory enough. The assumptions (normality, homoscedasticity, and independence) were not always respected and the performance criteria computed could be better. Other regression techniques could eventually be used to obtain a more satisfying model.

We undertook preliminary analysis using regression techniques (stepwise regression, ridge regression) to develop a new model. These techniques were not appropriate for the problem. The common regression assumptions were not respected. It was also noticed that the relation between the squat and the explanatory variables is very complex and is not linear. The regression tree technique was chosen because it is nonparametric and does not suppose a specific form for the relation between the squat and the explanatory variables. The results obtained with RegT are very promising: all performance criteria were better with these models (Table 7). Furthermore, we used a leave-one-out cross-validation to verify the quality of the predictions. The regression tree models gave honest squat predictions.

The RegT models are valid to predict the maximal squat of deep draft ships that navigate in the St. Lawrence Waterway or in waterways with similar characteristics. Nevertheless, the validity domain is extended by comparison to most empirical models. For example, the Eryuzlu 1 model was developed for bulk carriers with a block coefficient of 0.8, with a draught between 8.13 and 20.3 m and $1.1 < h/T < 3$. The Barrass 1 model can be used when $1.1 < h/T < 1.4$. In this work, the RegT models developed are valid for three types of ships, with a block coefficient that ranges from 0.6 to 0.9, a draught between 8.6 and 15.5 m, and $1.1 \leq h/T \leq 4.75$ (Tables 2–4). Other models could be adjusted with other data sets, from different waterways, using the same methodology. However, the applicability of the developed models to other situations could be verified with a new data set, for which the squat and its explanatory variables were measured. The predicted values obtained with the model could be compared with the measured squat values. Furthermore, the RegT models could be actualized with other data sets coming from other waterways to extend its validity domain.

From a practical point of view, this technique can be very useful. Once coded, the user only needs to enter the values of the explanatory variables and the corresponding squat prediction is given instantaneously by the prediction tree. With most empirical equations, the same independent variables are used. Thus, the model built with a regression tree is not more complicated and can be as handy as the empirical equations. Furthermore, it can be integrated into a dynamic underkeel clearance system. These systems combine accurate hydrographic data (charted depths water levels, currents) and ship-specific and channel-specific prediction formulas for squat, and give all information necessary to manage the underkeel clearance effectively and securely. The CCG is following this tendency with a project aiming to develop an integrated tool to manage the underkeel clearance of the maritime traffic on the St. Lawrence Waterway.

Acknowledgments

The writers wish to thank the Canadian Coast Guard (CCG) and the Natural Sciences and Engineering Research Council of Canada (NSERC) for funding this research. The writers would also like to thank the CCG for providing the high-quality database and Professor Karem Chokmani of INRS-ETE for his useful comments. The writers would also like to thank the editor, Professor Alexander Härting, and two anonymous reviewers for their valuable comments and suggestions.

Notation

The following symbols are used in this paper:

A_c = cross-sectional channel area (m^2);

A_s = ship underwater cross-sectional area (m^2);

A_w = net underwater channel cross-sectional area (m^2);
B = restricted channel width (m);
b = ship beam (m);
C_b = ship block coefficient;
E_{E1} = errors of the Eryuzlu 1 model;
F_{nh} = Froude number based on the undisturbed water depth;
$f(\cdot)$ = function of explanatory variables;
g = gravitational acceleration (9.81 m/s^2);
h = water depth (m);
K_b = width factor (function of W and b);
L = overall length of the ship (m);
L_{pp} = ship length between perpendiculars (m);
n = sample size;
PAR_h/T = water depth effect;
R^2 = coefficient of determination;
S = maximal squat measured (m);
$\hat{S}$ = predicted ship squat (m);
S_2 = channel blockage;
st = ship type (1=container ship, 2=tanker, 3=bulk carrier);
T = ship draft (m);
V = ship speed relative to the water (m/s);
V_{cr} = critical ship speed for unrestricted shallow water (m/s);
V_k = ship speed relative to the water (kn);
W = channel width (m);
$\mathbf{x}$ = matrix of explanatory variables;
$\mathbf{y}$ = vector of response values;
y_i = observed response value (m);
$\hat{y}_i$ = predicted response value (m);
β_0 = origin in the stepwise regression model;
β_i = coefficients of the explanatory variables in the stepwise regression model ($i=1,\ldots,17$);
ε = residuals in the stepwise regression model; and
σ^2 = variance of the residuals.

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